Employment protection and incentives: Severance pay vs. procedural inconvenience

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ABSTRACT

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I consider the effects of employment protection (EP) on worker incentives in the labor market with search friction, where EP is categorized into severance pay (SP) and procedural inconvenience (PI). When firms cannot distinguish shirkers, EP seems to negatively affect worker incentives, because shirkers are unlikely to be fired. However, EP can have a positive effect on worker incentives because diligent workers are protected by EP. It is shown that the positive effect can dominate the negative one when EP is moderate. In particular, PI tends to improve the unemployment rate and welfare, while SP has a somewhat unclear effect. J. Japanese Int. Economies 34 (2014) 272–290. Chuo University, Faculty of Commerce, 742-1, Higashinakano, Hachioji, Tokyo 192-0393, Japan.

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1. Introduction

This study examines the effects of employment protection (EP) on worker incentives in the labor market with search friction. OECD Employment Protection Indicators have shown that OECD countries have various rules of EP and degrees of EP strictness, and many researchers have drawn significant attention to the association of EP with economic performances. While numerous studies have focused on the effects of EP, relatively fewer studies have been conducted on the effects of EP on worker incentives. Worker incentives, however, are crucial for macroeconomic and corporate performances, and thus, the issue in this article should be considered.

There are negative and positive effects of EP on the worker incentives. Regarding negative effects, the efficiency wage model, in which threat of dismissal is a driving force in worker incentives, suggests that if EP deters firms from firing shirkers, EP has a disincentive effect. In the real world, however, even diligent workers who contribute to a firm’s success can be fired because of unfavorable business conditions. If workers are easily fired regardless of their contribution, they are discouraged from working hard for their firms. Thus, credible job security can encourage workers, and therefore, EP has a positive commitment effect on worker incentives. Accordingly, the effect of EP on worker incentives is complicated. This study considers the effects of EP when all fired workers, including shirkers, are protected by EP.

In this article, to analyze various rules of EP, I categorize EP into severance pay (SP) and procedural inconvenience (PI), which includes providing sufficient advance notice or negotiating with unions. Although both SP and PI increase firms’ firing costs, SP is a monetary transfer from firms to fired workers, whereas PI leads to a transaction cost. In that regard, it seems that SP is a better option than PI, because SP is not socially wasteful. However, SP may seriously damage worker incentives, because SP can benefit shirkers as well. This study also focuses on the difference between SP and PI.

While the disincentive effect of EP has often been realized, the commitment effect has been given less serious consideration except in some recent studies regarding skill formation. Suedekum and Ruehmann (2003) and Belot et al. (2007) indicate that introducing SP as a type of EP induces workers to make a human investment, because firms can promise workers the benefit of skill formation through SP. Booth and Zoega (2003) point out that firing policies tend to be excessively implemented, because firms are not concerned with human capital lost when workers quit and move to other industries. These studies show that if credible job security is provided, workers can receive compensation for their costs of skill formation, and thus, skill formation is promoted.

Although these previous studies generated insightful results, they were limited to analyses in partial equilibrium models. Even if the effect of EP is beneficial in partial equilibrium analyses, this may not be true for general equilibrium analyses. Thus, as shown in a recent study by Demougin and Helm (2011) in which they analyzed an incentive problem in the search model, it is crucial to consider the effect of EP on worker incentives from the perspective of general search equilibrium analyses.

To analyze whether the disincentive effect overrides the commitment effect, we must consider whether shirkers are protected by EP, especially whether they are eligible for SP. When EP provides benefits for only diligent employees, a negative effect on worker incentives from EP is unlikely. This is because if firms can verify shirkers, the shirkers do not have EP. Fella (2000) considers this situation with an efficiency wage model and shows that a positive effect is generated by SP.

Galdon-Sanchez and Guell (2003) analyze a different situation: shirkers are not protected by EP in principle, but third parties are unable to distinguish shirkers. In their model, firms can fire workers for their “shirking behavior” even if they do not shirk, and fired workers may not receive SP because of firms’ selfish behavior. When firms’ moral hazard is pervasive, firms can avoid incurring SP, and firing can be excessively implemented. Therefore, regulations that inhibit firms from easily firing workers provide a positive effect.

Fella (2000) considers a situation in which diligent workers are protected by EP, but shirkers are not. Galdon-Sanchez and Guell (2003) focus on a firm’s moral hazard in which workers cannot receive SP because
SP regardless of their behavior; that is, even diligent workers are not protected. In contrast, this article examines a different situation in which all fired employees, including shirkers, are protected by EP. In this situation, EP seems to be the cause of worker moral hazard. However, in this study, it is shown that the disincentive effect can be dominated by the commitment effect, and that EP has a positive effect on worker incentives.

While the degree to which diligent workers should be protected by EP has been controversial, most people agree that shirkers should be excluded from EP. Dismissals for shirking behavior, such as absence without notice or permission, are significantly different from dismissals caused by unfavorable business conditions, in which workers usually have no responsibility for poor corporate performance of their firms. This would be true if bad business conditions are caused by macroeconomic states, such as drastic changes of exchange rates or oil prices (which are not controlled by workers). However, workers' behavior can also influence a firm’s corporate performance. If workers efforts and contributions are poor, corporate performance is likely deficient, and the firm becomes unable to maintain employment. Shirking behaviors such as sabotage or absence without notice may be easy to verify, but there is a gray zone where it becomes more difficult to verify shirking behavior. For example, salespersons can enjoy tea time outside their companies, but their behaviors do not always come out. In this study, I pay considerable attention to reduced effort or poor contribution by workers (which are difficult to verify) rather than easily identifiable, obvious shirking behavior.

This study provides numerical illustrations explicitly showing the effects of SP and PI on the economy. The negative effects of SP on the unemployment rate are apparent, but PI tends to improve the unemployment rate. Furthermore, although both SP and PI increase firms’ firing costs, their effects on the unemployment rate contrast sharply when worker incentives is crucial and all workers are protected. While effects of SP are ambiguous depending on the matching situation, PI intensifies the commitment effect by encouraging workers to work hard and to improve social welfare. Additionally, equilibrium wage tends to be inflexible regardless of the level of EP.

The numerical illustrations indicate that an extreme relaxation in EP provisions or a rigorous EP damage worker incentives. This result is consistent with studies showing the positive effects of PI without explicit concerns regarding worker incentives, such as Ljungqvist and Sargent (1998, 2007), Rogerson and Schindler (2002), Pissarides (2001), and Blanchard and Tirole (2008). Part of this study's contribution is to show the positive effect generated by PI with the worker incentive problem.

The remainder of this paper is organized as follows. Section 2 presents a Mortensen–Pissarides matching model. In Section 3, an incentive compatible search equilibrium is derived. In Section 4, the effects of SP and PI are analyzed and numerical illustrations are shown. Finally, Section 5 presents the conclusions.

2. The model

2.1. Employment relations

I consider a standard Mortensen–Pissarides-type matching model. A firm with a job vacancy is randomly matched to an unemployed worker in the labor market. The matching function is given by \( m = m(u, v) \), where \( u \) is the unemployment rate and \( v \) is the vacancy rate. The vacancy–unemployment ratio indicating market tightness, \( v/u \), is denoted as \( \theta \). The matching function has constant returns to scale, that is, \( q(\theta) \equiv m(u/v, 1) \), where \( q(\theta) \) denotes the probability with which a job vacancy is matched to an unemployed worker. Clearly, \( q'(\theta) \leq 0 \).

In the labor market, all unemployed workers are considered identical regardless of their past behavior, because past behavior is not observed. Hence, the matching probability is equivalent among the unemployed. Similarly, all job vacancies in the market are identical for the unemployed.

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2 When the court makes no distinction between diligent workers and shirkers, legal disputes between workers and firms tend to occur. While the court may not be able to discern the absolute truth, it still must judge which side is “more” correct. Thus, workers and firms face uncertainty in legal disputes. The effect of EP tends to be unclear in uncertain situations. Stahler (2008) analyzes a search model with an uncertain situation on legal disputes and shows that the effect of SP is ambiguous. Additionally, while PI deters firms from firing and decreases market tightness, the effect of PI on the unemployment rate and welfare is not clear.
After matching, a worker must work hard to be highly productive at the effort cost \( c \). The firm cannot observe the worker’s effort but only his/her productivity. Productivity \( p \) of a worker who made an effort is stochastically distributed over the range \( p \in [0, +\infty) \). The density and distribution functions are denoted as \( \phi(p) \) and \( \Phi(p) \), respectively. When a worker shirks with zero effort cost, the density and distribution functions are given by \( \phi^S(p) \) and \( \Phi^S(p) \), respectively. Both functions \( \phi(p) \) and \( \phi^S(p) \) have full support on the range \( [0, +\infty) \), and the strict first-order stochastic dominance holds; that is, \( \Phi(0) < \Phi^S(0) \) for any positive \( p \) except \( p = 0 \), where \( \Phi(0) = \Phi^S(0) = 0 \) holds. This indicates that high productivity is more likely to be realized when a worker works hard than when she/he does not.

A wage is posted by a firm just after matching. The wage is not conditional on either the worker’s behavior or productivity \( p \); that is, a firm offers only a basic wage. This is similar to the efficiency wage model by Shapiro and Stiglitz (1984). The worker’s productivity is observable but unverifiable for the firm and the worker. Thus, firms cannot give incentives to workers through productivity-based wage schemes. The assumption on unverifiable productivity is crucial. If the worker’s productivity were verifiable by third parties, a firm could offer an incentive scheme contingent on the productivity, such as a piece rate. The incentive problem, therefore, could be solved through the incentive scheme.

The timing of the actions of a worker and a firm is as follows:

1. A firm is matched to a worker in the labor market and posts a wage \( w \).
2. The worker chooses whether or not to exert effort.\(^3\)
3. Productivity \( p \) of the worker is observed inside the firm.
4. The firm makes a decision regarding dismissal \( f \) but before producing output. If the worker is fired, she/he receives SP, is transferred to the unemployment pool, and receives the reservation wage \( w \). Otherwise, the worker produces output \( p \) and receives the basic wage \( w \).
5. In the next period, if the match is dissolved, the firm and the worker search for a new job match in the labor market. Otherwise, they repeat the same process: a new wage is offered by the firm, the worker chooses whether or not to exert effort, and the value of his/her productivity is stochastically determined. Productivity in each period is independent of those in past periods.

In this model, a wage is posted by a firm at the beginning of each period. Once it has been posted, it is inflexible during the incumbent period regardless of the worker’s productivity. In the next period, a new wage is offered. In this respect, wages are flexible at the beginning of each period but inflexible during a period.\(^4\) This assumption is reasonable, because firms often fire workers because their wages are inflexible and too high to maintain employment. Workers are fired involuntarily in this model. If wages were adjusted flexibly, firms would not have to fire workers, because workers would quit firms voluntarily when wages were lowered to a level below the reservation wage in the outside market.

### 2.2. Employment protection

When a firm fires a worker, the firm must incur a firing cost \( f \). Thus, the firm’s current profit is either \( p - w \) if the firm maintains the match or \(-f\) if the firm fires the worker. The firing cost \( f \) consists of two factors (\( s \) and \( z \)) given as \( f = s + z \), where \( s \) and \( z \) denote SP and PI, respectively. SP and PI are social rules exogenously imposed by the government, and SP is a lump-sum transfer that must be given to all fired workers including shirkers. If firms are allowed to deny SP to shirkers, firms may claim that all fired workers are “shirkers”, because a third party cannot verify who is shirking. If firms can do that in this model, EP would not work.

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\(^3\) If the worker rejects the posted wage, the worker remains unemployed and waits for a new matching opportunity in the next period. However, as mentioned later, firms offer a wage satisfying the participation constraint, and thus, workers accept it on the equilibrium path.

\(^4\) This is different from wage setting through a Nash bargaining model. Rocheteau (2001) shows that when the bargaining power of workers is not strong, their earnings through Nash bargaining are not sufficient to induce them to work hard. In this case, firms must post a high wage for worker incentives. Thus, we can apply this study to a situation with weak bargaining power. However, this study does not completely exclude a case with strong bargaining power, because high SP gives a high rent to a worker, similar to the strong bargaining power case.
2.3. Incentive compatibility

A worker is fired when his/her productivity is less than a threshold denoted as $\hat{p}$, which is endogenously determined. The threshold of dismissal is commonly applied for both diligent and shirking workers, because firms cannot observe what workers did. Even if a worker is diligent and sustains effort, she/he can be fired for low productivity with probability $\Phi(\hat{p})$. The fired worker then receives SP from the firm and enters the unemployment pool. Thus, the worker’s expected payoff in a period is given by $\Phi(\hat{p})(s + w) + (1 - \Phi(\hat{p}))w - c$. Similarly, a shirker is fired with probability $\Phi_S(\hat{p})$, and the expected payoff is represented by $\Phi_S(\hat{p})(s + w) + (1 - \Phi_S(\hat{p}))w$. Shirkers can therefore save the effort cost in a period. This is a benefit for shirkers although they are likely to lose their jobs in compensation for their shirking behavior. As moral hazard problems have been analyzed using discrete-time models in the literature of relational contracts and repeated games, I also consider a discrete-time model; thus, the benefit of shirking is easy to specify in discrete-time models.\footnote{In continuous-time models, when a worker shirks for a moment and works hard after that, the benefit of the shirking behavior is negligibly small because the worker can enjoy the benefit only for a moment. Thus, the benefit cannot be measured.}

The present discount value of the payoff of a diligent or a shirking worker is denoted as $E^N$ and $E^S$, respectively. The diligent worker’s payoff is given as

$$E^N = \Phi(\hat{p})(s + w) + (1 - \Phi(\hat{p}))w - c + \frac{1}{1 + r} \{ \Phi(\hat{p})U + (1 - \Phi(\hat{p}))E \},$$

where $E = \max\{E^N, E^S\}$, $r$ is the time preference rate, and $U$ is the present discount value of the payoff of the unemployed. Similarly, the present discount value of a shirking worker’s payoff is given by

$$E^S = \Phi_S(\hat{p})(s + w) + (1 - \Phi_S(\hat{p}))w + \frac{1}{1 + r} \{ \Phi_S(\hat{p})U + (1 - \Phi_S(\hat{p}))E \}.$$  

Finally, the present discount value of the unemployed is given as

$$U = w + \frac{1}{1 + r} \{ \theta q(\hat{\theta})E + (1 - \theta q(\hat{\theta}))U \}.$$  

These Bellman Eqs. (1)–(3) indicate that workers are risk-neutral. This assumption is not only for avoiding computational complexity but also for analysis on the effect of EP. If workers are risk-averse, they are more likely to prefer job security; thus, EP tends to provide a positive effect. Although I admit that risk aversion is realistic and crucial, in this study, I would like to eliminate the positive effect of EP through risk aversion of workers.

In this model, firms make a decision on firing after observing the productivity of workers and before producing outputs. Fired workers do not receive the basic wage and instead receive SP from their firms. I assume this model for the following reason. Suppose that fired workers receive a basic wage from their firms. Firms are then unwilling to fire workers, because once they have paid the basic wage, it is better for firms to produce output with their employed workers. Because productivity of workers is followed by the same probability distribution independently in each period, worker productivity does not depend on past events. Thus, firms do not fire workers after paying the basic wage.

The incentive compatible (IC) condition and individual rationality (IR) as the participation constraint are given by $E^N \geq E^S$ and $E \geq U$, respectively.\footnote{In this study, I consider the case in which a worker chooses two values for effort: Exerting effort or shirking. If the effort level of a worker is continuous, the incentive compatibility is given as the first-order condition of the worker’s optimal effort problem. Even if the mathematical representation of the model is different, the difference does not influence the results considerably, as continuous moral hazard models are similar to discrete ones.} To satisfy both, wage $w$ should be higher than the reservation wage $w$, that is, $w > w$. Otherwise, no worker would search for a new job or exert effort.

A shirker receives either the basic wage $w$ upon remaining at the firm or the reservation wage $w$ with SP upon being fired. On the other hand, an unemployed person receives only the reservation wage, $w$. Thus, $E^S > U$ holds from $w > w$. This also indicates that $E > U$ always holds, and thus, IR slack provided that $E^N > E^S$ holds. It is sufficient, therefore, to focus on the incentive compatibility $E^N \geq E^S$. The following condition is derived on the basis of $E^N \geq E^S$ and $E \geq U$:
When condition (4) is satisfied, both IC and IR hold. The derivation of Condition (4), which is called the IC condition throughout this paper, is given in the Appendix.

Threshold \( \bar{p} \) must be positive to maintain worker incentives, that is, \( \bar{p} > 0 \). If \( \bar{p} = 0 \) holds, workers would never be fired, and thus, the IC condition would never be satisfied. The absence of the threat of dismissal motivates workers to shirk.\(^7\)

As shown in the IC condition, an increase in SP is unlikely to meet the IC condition given a set of market tightness \( \theta \) and threshold of dismissal \( \bar{p} \). It seems that a hike in SP provides a negative effect on worker incentives. However, as shown later, SP influences the threshold of dismissal \( \bar{p} \). The effect of an increase in \( \bar{p} \) on the left-hand side (LHS) of the IC condition, \( F(w, \theta) \), is ambiguous. Thus, the effect of SP on worker incentives is complicated. In addition, the effect of PI is not clear. While PI (denoted as \( z \)) does not explicitly appear in the IC condition, PI also influences worker incentives indirectly through the threshold of dismissal. The effects of SP and PI on worker incentives, therefore, cannot be concluded by looking only at the IC condition.

In my later theoretical analysis, I indicate the relationship between wage and market tightness on the IC condition. As later shown, a firm posts an incentive compatible wage that satisfies the IC condition with equality as the IC curve in the \( \theta - w \) space in Fig. 1.

Wage determination is based on the IC curve in this model. In the textbook model, the wage determination curve trends upward: Wages increase with respect to market tightness, because a tight market is advantageous for workers’ wage bargaining.\(^8\) On the other hand, the IC curve may trend downward, because of the effect of market tightness on worker incentives is not clear. Additionally, while market tightness \( \theta \) influences the threshold of dismissal \( \bar{p} \), it is unclear how the threshold of dismissal is associated with the IC curve. Accordingly, the IC curve as wage determination may trend upward or downward.

### 2.4. Dismissal and job creation

The present discount value \( J \) of a firm matched to a worker is as follows:

\[
J = -\Phi(\bar{p})f + \int_{\bar{p}} (p - w)\phi(p)dp + \frac{1}{1 + \gamma} \{\Phi(\bar{p})V + (1 - \Phi(\bar{p}))J\}.
\]

The present discount value \( V \) of a job vacancy is \( V = -k + \frac{1}{1 + \gamma} \{q(\theta)f + (1 - q(\theta))V\} \), where \( k \) is the job vacancy cost. From the free entry and exit condition on job vacancies, \( V = 0 \). Hence, it holds that \( J = (1 + r)k/q(\theta) \). Denoting \( \bar{J} \) as \( \bar{J} \equiv J/(1 + r) \),

\[
\bar{J} \equiv \frac{1}{r + \Phi(\bar{p})} \left\{ \int_{\bar{p}} (p - w)\phi(p)dp - \Phi(\bar{p})f \right\} = \frac{k}{q(\theta)}.
\]

This is called the job creation (JC) condition in this paper. The JC condition means that the expected profit of a matched firm is equivalent to the average vacancy cost.

If a firm fires a worker, the present discount profit is \(-f\) given that \( V = 0 \). On the other hand, if a firm maintains the match, the present discount profit is given by \( p - w + \bar{J} = p - w + k/q(\theta) \). Hence, the threshold of dismissal is given by

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\(^7\) Even a shirker can produce a positive output in this model although there is little likelihood of high productivity. Thus, depending on the parameters, a firm may have no incentive to offer the incentive compatible wage satisfying the IC condition. In this study, it is assumed that a firm is willing to offer the incentive compatible wage to maximize its profit. Otherwise, worker incentives are not crucial, and the problem considered in this study disappears.

\(^8\) It is denoted as the “wage curve” in Pissarides (2000, p. 19).
\[ p = \frac{w}{C_0} f / C_0 k = q(h) \]  

(6)

When a worker’s productivity is over \( p \), a firm upholds the match. As shown in (6), a reduction in the wage or an increase in the firing cost reduces the threshold \( \hat{p} \). In addition, a firm hesitates to fire a worker under high vacancy cost or market tightness.

In a similar manner to the IC curve, I consider the relationship between \( w \) and \( \theta \) on the JC condition (5):

\[ G(w, \theta) = \frac{\hat{f}}{q(\theta)} - k = 0 \]

This is represented as the JC curve in Fig. 1. The JC curve in this model is similar to Fig. 1a. \( E_A \) is a stable equilibrium.

Fig. 1b. \( E_B \) is not stable.

Fig. 1c. \( E_C \) is a stable equilibrium.

\[ \hat{p} = w - f - k/q(\theta) \]  

(6)
to the textbook model. If matched firms and vacancies confront a wage hike, the expected profit of a matched firm decreases. To maintain the zero profit of a vacancy as an equilibrium condition, market tightness must decrease. The JC curve, therefore, trends downward on the $\theta - w$ space.

3. Equilibrium

3.1. Incentive compatible wage

Here, I consider the posted wage. Although the threshold is determined by the firm after the wage has been posted and the effort cost has been sunk, the worker and the firm can anticipate threshold $p$ upon being matched. An individual firm is too small to affect market tightness in the labor market, and thus, an individual firm offers a wage given a level of market tightness that is determined in the labor market. If a firm offers a sufficiently low wage (e.g., $w = 0$), the IC condition does not hold for any $\theta$: The LHS of the IC condition is not positive from the first-order stochastic dominance, that is, $F(w, \theta) \leq 0$. Thus, the posted wage must be sufficiently high to satisfy the IC condition. If we consider reasonably continuous matching and density functions, the LHS of the IC condition is continuous with respect to $w$ given a level of market tightness. When there is a range of wages satisfying the IC condition $F(w, \theta) \geq c$, we can find a wage satisfying the IC condition with equality $F(w, \theta) = c$ from the intermediate value theorem as $w$ increases from 0. The firm is willing to minimize the wage as long as the IC condition is satisfied, and thus, the posted wage $w^*$ is the minimum value among the wages satisfying $F(w, \theta) = c$ given $\theta$. This means that the IC condition is binding at the equilibrium, and the result is given as a lemma:

**Lemma 1.** When there are wages satisfying the IC condition given a level of market tightness $\theta$, the posted wage is the minimum value satisfying $F(w, \theta) = c$ under the market tightness.

The proof is provided in the Appendix. Generally, there may not be an incentive compatible wage depending on the parameters, but it is not unusual that the IC condition can be satisfied given a level of market tightness. For example, when $s$ is low, $F(w, \theta)$ is positive under $w > w$. Thus, the IC condition holds under the market tightness if the effort cost $c$ is not sufficiently high.

3.2. Search equilibrium

I have shown the posted wage given a level of market tightness from the IC condition. In this model, a search equilibrium $(\theta, w)$ is characterized using the IC condition (4) and the JC condition (5) along with the threshold (6). Thus, an equilibrium is represented at the intersection of the IC and JC curves in Fig. 1.

As previously mentioned, the association between the incentive compatible wage and market tightness on the IC condition is complex. As shown in the incentive compatible condition (4), a hike of the threshold of dismissal can increase or decrease the incentive compatible wage, because high job security has two effects (i.e., the disincentive and commitment effects). Market tightness also influences the threshold of dismissal and the incentive compatible wage on the IC condition, and the effect is complex. Accordingly, the shape of the IC curve is not globally obvious depending on the parameters. The IC curve may trend upward or downward; thus, there are three types of curve intersections locally, $E_A$, $E_B$, and $E_C$, as shown in Figs. 1a–c, respectively.

The difference between the JC and IC slopes is crucial for stationarity of equilibrium. I focus on the stable search equilibria throughout this paper. Point $E_A$ in Fig. 1a is a stable equilibrium, but $E_B$ in Fig. 1b is not stable because a new entry can occur at this point. Suppose some new vacant firms enter the market at point $E_B$. In this case, the market tightness increases from $\theta_3$ to $\theta_4$ in Fig. 1b. Under the new market tightness $\theta_4$, a new minimum incentive compatible wage is given as $w_4$ on the IC curve, which is lower than the original wage $w_3$. The point $(\theta_4, w_4)$ is below the JC curve, and thus, the expected profit of a vacancy is positive and greater than that at point $E_B$ where the vacancy’s profit is zero. New vacancies are expected to generate positive profit, and thus, they enter the market. Point $E_B$ is not a stable equilibrium.
In contrast, a new entry never occurs at point $E_A$. If new vacancies enter the market at point $E_A$, market tightness increases from $\theta_1$ to $\theta_2$ in Fig. 1a. Under the new market tightness $\theta_2$, a new minimum incentive compatible wage is given as $w_2$. The point $(\theta_2, w_2)$ is above the JC curve, and the vacancy’s profit is negative. Because a new entry does not occur, point $E_A$ is a stable equilibrium, where the slope of the IC curve is locally mathematically greater than that of the JC curve.

Although the IC condition is represented as a downward-sloping curve in Figs. 1a and 1b, the IC curve may trend upward in the $\theta - w$ space (as shown in Fig. 1c). In this case, similar to the case shown in Fig. 1a, the slope of the IC curve is mathematically larger than that of the JC curve. Then, the same explanation previously mentioned applies to this case.

3.3. Equilibrium unemployment rate

Next, the equilibrium unemployment rate is derived. The total labor workforce is normalized to 1, and inflow to and outflow from unemployment are then given by $(1 - u)\Phi(\hat{p})$ and $\partial q(\theta)u$, respectively. From the steady-state condition, in terms of job flow, inflow should be equivalent to the outflow. Thus, the equilibrium unemployment rate is as follows:

$$ u = \Phi(\hat{p})/\{\Phi(\hat{p}) + \partial q(\theta)u\}. \quad (7) $$

The unemployment rate is specified using the threshold of dismissal and the market tightness. This is known as the Beveridge curve.

4. Effect of employment protection

4.1. A case of constant firing costs

The model was setup and the search equilibrium was derived to investigate the effects of EP. First, I consider how the change in the relative share of SP affects the economy given a constant level of firing cost, that is, $f = \hat{f}$.

**Proposition 1.** When the relative share of SP increases given a constant level $\hat{f}$ of the firing cost, it raises the incentive compatible wage and reduces market tightness. Concurrently, workers are likely to be fired and the unemployment rate increases.

The proof is provided in the Appendix. Proposition 1 shows that a high relative share of SP damages worker incentives and the economy under a constant firing cost $f = \hat{f}$. Because a shirker is entitled to receive SP whenever she/he is fired, an increase in the share of SP discourages a worker from making the required effort under $f = \hat{f}$. A high wage offer with a high threshold of dismissal is therefore needed to maintain worker incentives, reducing the firm’s profit from the match. This induces vacancies to exit the market, reducing market tightness. Hence, the unemployment rate increases because of the high threshold of dismissal and low market tightness, as shown in (7). In contrast, if the share of PI increases, the opposite result occurs. That is, wage decreases and market tightness increases, with a decrease in the threshold of dismissal and the unemployment rate.

The result of Proposition 1 is explained graphically as follows. When the total firing cost does not change, the JC curve does not shift in the $\theta - w$ space as in Fig. 2a. On the other hand, an increase in the share of SP shifts the IC curve upward in the $\theta - w$ space, thereby increasing the wage and reducing the market tightness.

Next, we consider the $u - v$ space in Fig. 2b. The job destruction rate $\Phi(\hat{p})$ depends on threshold $\hat{p}$, which is raised by the increase in wage $w$ and the reduction in market tightness $\theta$. A high relative share of SP therefore increases the job destruction rate, resulting in an increase in the unemployment rate. Thus, the Beveridge curve (7) shifts upward in the $u - v$ space because of the increased job destruction rate $\Phi(\hat{p})$, as shown in Fig. 2b. In addition, the market tightness (MT) curve in Fig. 2b shifts clockwise because

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9 In general, in matching models with endogenous job destruction, the Beveridge curve does not always curve downward. However, in this model, it is easy to show a downward-trending Beveridge curve.
of the reduction in market tightness $\theta$, where the MT curve in Fig. 2b represents the equilibrium market tightness given by the intersection of the IC and JC curves in Fig. 2a. An increase in the share of SP shifts the equilibrium accordingly from $E_1$ to $E_2$ in Fig. 2b. The unemployment rate unambiguously increases.

4.2. Numerical illustration

Proposition 1 holds true only when the total firing cost is fixed. This situation may be restrictive. As is often the case with the matching model, this model is too complex to provide explicit analytical results on the effect of EP. Therefore, I provide numerical illustrations. In the numerical illustrations, it is assumed that the productivity of a diligent worker is normally distributed with mean 100 and standard deviation 35. Similarly, the productivity of a shirker is normally distributed with mean 45 and the same standard deviation. The worker productivity distributions satisfy the first-order stochastic dominance.

The matching function is given as $m(u, v) = 0.7u^{z}v^{1-z}$, where the elasticity of matching with respect to the unemployment rate is denoted as $z$. As indicated by Pissarides (2000) and Cahuc and Zylberberg (2004), the elasticity of matching has been estimated to range from 0.4 to 0.7. Thus, numerical illustrations are provided for both $z = 0.4$ and $z = 0.7$. 

\[ \text{Fig. 2a. Search equilibrium.} \]

\[ \text{Fig. 2b. Search equilibrium.} \]
Other parameters are set as follows: \( w = 50, r = 0.0125, c = 25, \) and \( k = 10. \) The reservation wage is half of the mean value of productivity that a worker generates for his/her firm. I consider a shock for one quarter annually; thus, the time discount rate is \( r = 0.0125, \) which is approximately equivalent to 5\% annually. The effort cost \( c \) of a worker is unobservable in the real world. If \( c \) is high, the benefit of the worker effort is negligible, because it is too expensive to maintain worker incentives. In contrast, if \( c \) is small, a firm does not have to seriously consider worker incentives. Thus, worker incentives is a critical problem when the effort cost is intermediate. In this simulation, the effort cost is given as half of the reservation wage. Finally, the vacancy cost \( k \) is the recruitment cost for a vacancy and is chosen to fit the unemployment rate using a value reasonably compatible with the real world.

4.3. Market tightness, wage, and threshold of dismissal

As I consider a discrete-time model, both the probabilities \( \theta q(\theta) \) (with which an unemployed worker gets a new job) and \( q(\theta) \) (with which a vacancy is matched to an unemployed worker) must be equal to or smaller than 1. From the matching function, the probability conditions \( \theta q(\theta) \leq 1 \) and \( q(\theta) \leq 1 \) generate approximately \( 0.410 \leq \theta \leq 1.812 \) under \( \alpha = 0.4 \) and \( 0.601 \leq \theta \leq 3.284 \) under \( \alpha = 0.7. \)

The market tightness derived using the numerical illustrations is shown in Fig. 3, where the bold parts of the curves satisfy these probability conditions. Similarly, as later shown, variables in the numerical illustrations, such as wage, threshold of dismissal, and unemployment rate are represented by the bold parts when they satisfy these probability conditions.

Remark 1. The numerical illustrations indicate the following results:

(1) Fig. 3 shows that a hike of SP or PI decreases market tightness.
(2) Figs. 4 and 5 show that the effects of SP on the IC wage and the threshold of dismissal are ambiguous given a level of PI. A hike of SP induces firms to fire workers in the case of \( \alpha = 0.7 \) (i.e., high elasticity of matching with respect to the unemployment rate) but not in the case of \( \alpha = 0.4. \)
(3) An increase in PI lowers the IC wage and the threshold of dismissal given a level of SP; therefore, workers are unlikely to be fired.
(4) While a hike of SP or PI does not appreciably influence the IC wage, the effects of SP or PI appear through the likelihood of dismissal.

Both SP and PI decrease market tightness. This result differs from the effects created by the relative change in the share of SP and PI as shown in Proposition 1 (where an increase in the share of SP deters vacancies from remaining in the market, but an increase in the share of PI induces vacancies to enter the market). Remark 1(1) indicates that a firing cost hike induces firms to leave the market regardless of SP and PI.

![Fig. 3. Market tightness. The bold parts of the curves satisfy the conditions that the probabilities of a firm and a worker to be matched are not greater than 1.](image-url)
An SP hike has two opposite effects on worker incentives. As indicated by Remark 1(1), an SP hike decreases market tightness. This means that SP induces vacancies to leave the market; this is a threat for workers, because workers are unlikely to get a new job if they are fired. This is a positive effect of SP through market tightness, that is, encouraging workers to work hard in the hope of remaining in their firms. However, SP is earnings for fired workers, including shirkers. This is a negative effect on worker incentives, because they may not fear losing their jobs.

These two opposite effects complicate the way SP affects the equilibrium wage and the threshold of dismissal. It is easier for fired workers to get new jobs under a high elasticity of matching given a market tightness. Thus, the effect of SP depends on the elasticity of matching: The positive effect of SP on worker incentives is likely to be dominated by the negative effect in the case of high elasticity of matching, because an SP hike tends to damage worker incentives under a high elasticity of matching. If SP negatively affects worker incentives, a higher wage and/or a higher threshold of dismissal are required to maintain worker incentives. Actually, in the case of $\alpha = 0.7$ shown in Figs. 4b and 5b, an SP hike tends to increase the incentive compatible wage and the threshold of dismissal. In contrast, in the case of $\alpha = 0.4$ shown in Figs. 4a and 5a, the wage and threshold are likely to be negatively associated with SP.

In contrast, increasing PI positively affects worker incentives as a whole. As shown in Fig. 3, PI decreases market tightness in a manner similar to SP. This effect encourages workers to work hard. Furthermore, PI does not provide earnings for fired workers. The threat of losing their jobs is still a crucial factor for workers. Thus, a high wage offer with a high threshold of dismissal is not required to maintain worker incentives. The incentive compatible wage and the threshold of dismissal decrease with PI, as shown in Figs. 4 and 5. This effect appears regardless of the elasticity of matching. Therefore, PI is less likely than SP to inflict damage on worker incentives.
Fig. 4 shows that the quantitative effects of SP or PI on the incentive compatible wage is slight. The difference in wages caused by an SP or PI hike is less than 1%. When SP or PI increases by 20–30% compared to the average productivity of workers, the effect of SP or PI on the wage is less than 1% on average productivity. On the other hand, when $\alpha = 0.7$, a worker is fired with probability 7.3% when SP = 20 and PI = 0 and with probability 3.9% when SP = 20 and PI = 30; that is, workers are 3.4 percentage points more likely to be fired under the former case. The likelihood of dismissal increases by 87.2% (i.e., 3.4/3.9). This is similar in the case of $\alpha = 0.4$. While the effect of SP or PI is more likely to appear through the threshold of dismissal, an SP or PI hike does not substantially influence the incentive compatible wage level.

4.4. Unemployment rate

Remark 2. As shown in Fig. 6, the unemployment rate tends to increase with SP but decrease with PI. As shown in Fig. 3, a PI hike decreases market tightness, the effect of which increases the unemployment rate. However, PI has the opposite effect on the unemployment rate through the threshold of dismissal. As shown in Fig. 5, an increase in PI lowers the threshold of dismissal, which causes a lower unemployment rate. Although PI has both of these effects, Remark 2 indicates that the effect of PI on the unemployment rate through the threshold of dismissal dominates its effect through market tightness, and the unemployment rate tends to decrease with PI regardless of the elasticity of matching.

While a hike of SP provides an ambiguous effect on the IC wage and the threshold of dismissal (as indicated in Remark 1(2)), the unemployment rate is apt to increase with SP (as shown in Fig. 6). SP can also influence the unemployment rate through two routes. One route is through market tightness similar to that of PI: An SP hike reduces market tightness, which can raise the unemployment rate. The other route is through the threshold of dismissal depending on the elasticity of matching, as shown in Fig. 5a and b. If an SP hike increases the threshold of dismissal as in Fig. 5b, the unemployment rate increases. In this case, the unemployment rate increases with respect to SP, because the two routes of influence increase the unemployment rate.

Even if an SP hike decreases the threshold (as in Fig. 5a), which helps to decrease the unemployment rate, the entire effect of SP tends to increase the unemployment rate. In this case, SP can increase the unemployment rate through market tightness but also decrease it through the threshold of dismissal. Although this is similar to PI, Fig. 6 shows that the entire effect of SP on the unemployment rate is opposite that of PI. While SP or PI raises the firing cost for firms, the effect of each on the unemployment rate is likely completely opposite.

The quantitative effect of PI decreases the unemployment rate significantly. In the case of $\alpha = 0.7$, given SP = 20, the unemployment rates under PI = 0 and PI = 30 are 6.9% and 5.7%, respectively; the gap in the unemployment rate here is approximately 1.2 percentage points. When the total firing cost is
the same, the effect of PI is much larger. The unemployment rates are 4.1% under PI = 30 and SP = 10 and 9.4% under PI = 10 and SP = 30, a difference of 5.3 percentage points. The result is similar when \( \alpha = 0.4 \).

In this model, the inflow to unemployment, which is equivalent to the outflow from unemployment at equilibrium, is given by the likelihood of dismissal \( \Phi(p) \). As shown in Fig. 5, an increase in PI lowers the inflow to unemployment by reducing the threshold of dismissal. This indicates that PI has a negative effect on the reallocation of the labor force, which is similar to results in employment protection literature. The numerical illustrations showing that the speed of the inflow to and outflow from unemployment is not significantly associated with the unemployment rate are also consistent with the literature.

4.5. Social welfare

As the same situation is repeated in every period on the equilibrium path, we can focus on social welfare within a period. Social welfare within a period is given by

\[
\Omega \equiv \left\{ \int_{\bar{p}} p \phi(p) dp + \Phi(\bar{p})(w - z) - c \right\} (1 - u) + uw - k\delta u. \tag{8}
\]

The first term indicates welfare generated by all matches. At the equilibrium, the fraction of employed workers, \( 1 - u \), bears the effort cost and then produces output in the productivity range \( p \geq \bar{p} \). When productivity is less than \( \bar{p} \), the workers are fired and receive the reservation wage \( w \). They are fired with probability \( \Phi(\bar{p}) \); thus, PI is deducted as a transaction cost in terms of social welfare. SP disappears as it is only a monetary transfer from firms to workers. The second and third terms indicate contributions to welfare by unmatched players, the unemployed, and vacancies. When the unemployed and vacancies stay in the market, the unemployed receive the reservation wage but vacancies incur the vacancy cost.

Remark 3. As shown in Fig. 7, the effect of SP on welfare is ambiguous given a level of PI. That is, an SP hike improves social welfare in the case of \( \alpha = 0.4 \) but not in the case of \( \alpha = 0.7 \). Social welfare, however, tends to be positively associated with PI.

As shown in Fig. 7, the social welfare curves first worsen gradually with respect to a low SP and then improve. Within the range that the probability condition holds, SP is likely to improve social welfare in Fig. 7a but worsen it in Fig. 7b, showing that the effect of SP on welfare is ambiguous. When the incentive compatible wage and the threshold of dismissal increase with SP (as shown when \( \alpha = 0.7 \)), SP has a negative effect on worker incentives and worsens welfare. In contrast, SP may improve welfare when SP decreases the incentive compatible wage and the threshold of dismissal, such as when \( \alpha = 0.4 \).

![Fig. 7. Welfare.](image-url)
PI tends to improve social welfare regardless of the elasticity of matching. Because PI is socially wasteful as a transaction cost whereas SP is a monetary transfer, we may expect that PI, rather than SP, has a negative effect. However, we see the opposite result.

To summarize Remarks 2 and 3, while the effect of SP on welfare is ambiguous, SP tends to increase the unemployment rate and PI is likely to reduce the unemployment rate and improve welfare. From the perspective of economic policy, PI has a more significant role than SP when all workers are protected and worker incentives are crucial. However, this does not mean that only PI is essential and SP is not required, because SP can improve welfare depending on the matching situation.

4.6. Long-term shock

As previously stated, I used a quarter-term shock annually at a discount rate of 0.0125. A worker is unlikely to be fired if the downturn continues only for a quarter and economic conditions soon improve. A firm is inclined to continue employing a worker if low productivity is expected for only a short period. In contrast, if low productivity persists for considerably longer, a firm may be unwilling to retain the match, and the match is likely to be dissolved.

To examine a long-term productivity shock, I implement numerical illustrations with a higher discount rate, \( r = 0.1 \), indicating that the low productivity situation continues for approximately 2–4 years. If we regard the annual discount rate as 5%, a discount rate \( r = 0.1 \) shows that a productivity shock continues for 2 years. Recently, very low interest rates have appeared in developed countries after the global financial crisis in 2008. If we believe that the annual rate compatible with this recent financial situation is much lower than 5%, the rate \( r = 0.1 \) would indicate a period much longer than 2 years.
A high discount rate raises the threshold of dismissal as well as the unemployment rate. The numerical illustrations, however, show that PI reduces the unemployment rate in the case of $r = 0.1$ to a greater extent than in the case of $r = 0.0125$. In Fig. 8, the difference in the unemployment rate between $z = 10$ and $z = 0$ is denoted as $\Delta u \equiv u_{z=10} - u_{z=0}$. The $\Delta u$ curves decrease with SP in Fig. 8a and b. The $\Delta u$ curve under $r = 0.1$ is always located below the curve under $r = 0.0125$ regardless of the elasticity of matching. This shows that to reduce the unemployment rate, PI is more effective in the case of $r = 0.1$ than in the case of $r = 0.0125$.

Similarly, the difference in social welfare is denoted as $\Delta \Omega \equiv \Omega_{z=10} - \Omega_{z=0}$. To compare the effects of PI under the long-term or short-term shocks, the difference is normalized using the welfare level with $z = 0$: $\Delta \Omega/\Omega_{z=0}$. As shown in Fig. 9, the social-welfare-improving effect is larger in the case of $r = 0.1$ than in the case of $r = 0.0125$. This result is not influenced by the elasticity of matching. Thus, the positive effect of PI is not necessarily smaller when low productivity persists for a long period.

5. Conclusion

I have analyzed the effects of SP and PI on worker incentives in the labor market with search friction when all workers including shirkers are protected by EP. This study has shown that the effect of PI on worker incentives is likely to be positive compared with the effect of SP and that PI plays a significant role in worker incentives. Specifically, (1) both SP and PI decrease market tightness; that is, they induce vacancies to leave the market. (2) While the effects of SP on the incentive compatible wage, the threshold of dismissal, and welfare are ambiguous, SP tends to negatively affect the unemployment rate. (3) PI is likely to decrease the incentive compatible wage, the threshold of dismissal, and the unemployment rate, and it has a positive effect on social welfare. (4) A hike of SP or PI does not substantially affect the incentive compatible wage; however, this effect mainly appears through the threshold of dismissal and market tightness. (5) An increase in PI lowers the inflow to and outflow from unemployment. Consistent with the literature, the speed of this inflow and outflow is not significantly associated with the unemployment rate.

Accordingly, PI is more effective than SP when worker incentives are crucial and every worker is protected by EP; thus, it is important to provide sufficient notice and to negotiate with unions. Although EP seems to negatively influence worker incentives when shirkers are also protected, the conclusion of this study indicates that this is not always the case. However, I do not imply that we should have stringent EP. The main message is that EP, especially PI, can positively influence worker incentives when it is moderate.

Some points of this study may be controversial. First, we may need to consider search externalities. Because social welfare may be associated with search externalities, this positive effect on social welfare (as mentioned in Remark 3) may be due to the elimination of negative search externalities. Social welfare is affected through various paths and by multiple factors in this model, and it is too difficult to evaluate each path explicitly. Although estimating how search externalities are associated with social welfare in this model is complex, I can infer the effect of the search externalities to some extent.

In the textbook model of Pissarides (2000) with wage (Nash) bargaining but with no incentive problem and no firing cost, the optimal level of bargaining power is uniquely determined using the Hosios (1990) condition; that is, it is efficient when workers’ bargaining power is equivalent to the elasticity of matching. When the bargaining power of workers is weak compared to the elasticity of matching, as shown by the Hosios condition, the unemployment rate is excessively low from the perspective of efficiency. In this case, social welfare improves if a policy raises the unemployment rate. We start with weak bargaining power of a worker. As the bargaining power strengthens, social welfare initially improves until the bargaining power exceeds the elasticity of matching, at which point it declines.

Although this model with the incentive problem and firing cost differs from the textbook model, the effect of SP may be similar to that of workers’ bargaining power because high SP can increase the basic wage. Actually, the numerical illustrations show that the unemployment rate and the incentive compatible wage increase as SP increases from 0 when both $\alpha = 0.4$ and $\alpha = 0.7$. This is consistent with the fact that strong bargaining power of a worker increases the wage and the unemployment rate.
in the textbook model. If search externalities are crucial only to social welfare in this model, the social welfare curve in Fig. 7 should have an inverted U-shape with respect to SP showing that social welfare first increases then decreases with respect to SP in the case of \( z = 0 \). In contrast, as shown in Fig. 7, social welfare decreases as SP increases from 0. This implies that the improvement in social welfare generated by PI is not caused only by the elimination of search externalities. The effects of search externalities, therefore, are not essential to the result of this study.

Next, this model assumes that a random shock to productivity is repeated independently from the identical distribution. Thus, we can avoid complexity of analysis. However, as a vintage model suggests, the productivity of a match may initially be high and then decrease. In this case, long-term employment relations become inefficient, and the positive effects of EP shown in this study are alleviated. From the perspective of the vintage model, EP should be more stringent in the beginning of matching but more moderate for a long-term match. However, in contrast, we see that a long-term match may be more productive because of workers’ skill formation and cooperative industrial relationships, which are difficult to establish in a short period. In this situation, EP may have a more positive effect. Accordingly, the effects of EP hinge on productivity of matching which may vary over time. Although this viewpoint is outside the scope of this paper, it is a crucial issue left for future analysis, especially through empirical perspectives.

Finally, I have not considered the difference of wage-setting between wage posting and wage bargaining. Between permanent employees and fixed-term ones in developed countries, there are earnings and working condition gaps. The difference of wage setting may be associated with this gap. A recent study by Masui (2011) shows that the proportion of matches under wage bargaining is lower than the socially optimal level if no employment policy is implemented. The study implies the need for analyzing the effects of EP under each wage-setting, and that is another issue for future work.

Appendix A

A.1. Derivation of the IC condition (4)

When the incentive compatibility, \( E^N \geq E^S \), holds, it is obvious that \( E = E^N \), and we can substitute (3) into (1):

\[
E^N = \{ \Phi(s + \ddot{w}) + (1 - \Phi)w - c\} + \frac{\Phi\ddot{w}}{r + \theta q} + \frac{E^N}{1 + r} - \frac{1}{1 + r} \frac{r\Phi E^N}{r + \theta q}.
\]

Similarly, using \( E = E^N \), substitute (3) into (2):

\[
E^S = \{ \Phi^S(s + \ddot{w}) + (1 - \Phi^S)w\} + \frac{\Phi^S\ddot{w}}{r + \theta q} + \frac{E^N}{1 + r} - \frac{1}{1 + r} \frac{r\Phi^S E^N}{r + \theta q}.
\]

From (A1) and (A2), the incentive compatibility is replaced as

\[
E^N - E^S = (w - s - \ddot{w})(\Phi^S - \Phi) - c - \frac{\ddot{w}(\Phi^S - \Phi)}{r + \theta q} + \frac{\Phi^S - \Phi}{1 + r} \frac{rE^N}{r + \theta q} \geq 0.
\]

We solve (A1) with respect to \( E^N \), and derive the following equation:

\[
\frac{1}{1 + r} \frac{rE^N}{r + \theta q} = \frac{1}{r + \theta q + \Phi} \left\{ \Phi s + (1 - \Phi)w - c + \Phi w \frac{1 + r + \theta q}{r + \theta q} \right\}.
\]

W substitute (A4) into the middle hand side of (A3), then calculate and derive the IC condition (4).

A.2. Proof of Lemma 1

An individual firm is too small to affect market tightness, thus, firms offer wages given the market tightness determined in the labor market. As (6) implies, the threshold of dismissal, \( \tilde{p} \), will be zero if \( w \leq f + k/q(\theta) \). Using \( \Phi^S(0) = \Phi(0) = 0, F(w, \theta) = 0 \) holds in this case. When a wage is a little greater
than $f + kq(\theta) + \rho > 0$ holds, and $\Phi^5(\hat{\rho}) - \Phi(\hat{\rho})$ is positive. However, if $w \leq w_0$ holds at the same time, the numerator in the first bracket in the LHS of the IC condition is negative: $F(w, \theta) < 0$. Furthermore, when $w > \max\{f + kq(\theta), w_0\}$ holds, the LHS of the IC condition is positive. From the above statement, the value of $F(w, \theta)$ is near 0 or negative when $w$ is low, and it can be positive as $w$ increases. As $w$ increases, the IC condition can be satisfied under the market tightness.

$F(w, \theta)$ is continuous under the market tightness with respect to $w$ when the matching and density functions are smoothly continuous. If there is any range of wages satisfying the IC condition, $F(w, \theta) > c$; $F(w, \theta) < c$ holds when $w$ is low, and $F(w, \theta) > c$ holds as $w$ increases. Using the intermediate value theorem, there is a wage satisfying $F(w, \theta) = c$ as $w$ increases from 0. The wage is the minimum value satisfying the IC condition. Since the firm is willing to minimize the wage as long as the IC condition holds, the minimum value is posted as the wage.

A.3. Proof of Proposition 1

(1) I proceed to derive the mathematical condition on stable equilibria. At the minimum incentive-compatible wage denoted as $w^*$, $F(w^*, \theta) = c$ holds. From the continuation of $F(w, \theta)$, it holds that $F(w^* - \varepsilon, \theta) < c < F(w^* + \varepsilon, \theta)$ for a sufficiently small and positive $\varepsilon$. This shows that the curve of $F(w, \theta)$ is upward with respect to $w$ at $w = w^*$. This is mathematically represented as $F_w = \frac{\partial F(w, \theta)}{\partial w} > 0$. In other words, $F_w < 0$ means that a firm can reduce the wage further at the equilibrium while the IC condition is satisfied, thus, increasing the firm’s profit. This contradicts the original wage maximizing the firm’s profit. Hence, $F_w > 0$ holds at the equilibrium.

The equilibrium is specified by $F(w, \theta) = c$ and $G(w, \theta) = 0$. Differentiating them with respect to $w$ and $\theta$, the slope of the IC and JC curves are derived: $\frac{dw}{ds} = \frac{-F_r}{F_w}$ and $\frac{d\theta}{ds} = \frac{-G_r}{G_w}$, where $F_x \equiv \frac{\partial F}{\partial x}$ and $G_x \equiv \frac{\partial G}{\partial x}$ for $x = w$ or $\theta$. Since the slope of the IC curve is mathematically greater than that of the JC curve at the equilibrium, it holds that $-\frac{F_{r_w}}{F_w} > -\frac{G_{r_w}}{G_w}$. Using $G_w \equiv J_w = -\frac{1}{\tau} \{\frac{\partial F(p)}{\partial p}\} < 0$, $G_{\theta} = \frac{kq}{q^2} < 0$, and $F_w > 0$, the above inequality is replaced as

$$F_w = \frac{kq}{q^2} - F_{\theta J_w} < 0. \quad (A5)$$

(2) Next I conduct comparative statics regarding the search equilibrium with respect to $s$ subject to $df = ds + dz = 0$:

$$
\begin{align*}
\left(\frac{dw}{ds}\right) & = \left(\frac{G_w}{G_0} - \frac{F_0}{G_0}\right) \left(\frac{F_s}{J_s}\right), \\
\left(\frac{d\theta}{ds}\right) & = 1 \left(\frac{F_w}{q^2} - F_{\theta J_w} \left(\frac{kq}{q^2} - F_\theta \right) \left(\frac{F_s}{J_s}\right) \right), \quad (A6)
\end{align*}
$$

where $F_x \equiv \frac{\partial F}{\partial x} = -\frac{\tau + kq(\theta)}{\tau + kq(\theta)} \{\Phi(\hat{p}) - \Phi(\hat{\rho})\} < 0$. Note that $J_s = \frac{\partial J_s}{\partial s} = \frac{\partial J_s}{\partial \rho} = 0$ under $df = ds + dz = 0$. Thus, (A6) becomes

$$
\begin{align*}
\left(\frac{dw}{ds}\right) & = 1 \left(\frac{F_w}{q^2} - F_{\theta J_w} \left(\frac{kq}{q^2} - F_\theta \right) \left(\frac{F_s}{J_s}\right) \right), \\
\end{align*}
$$

Using (A5) and $J_w = -\frac{1}{\tau + kq(\theta)} < 0$, it is derived that $\frac{d\bar{w}}{df} |_{f-f} > 0$ and $\frac{d\bar{w}}{df} |_{f-f} < 0$. The results lead to $\frac{d\bar{w}}{df} |_{f-f} > 0$ from (6), thus, it holds from (7) that $\frac{d\bar{w}}{df} |_{f-f} > 0$.

References